

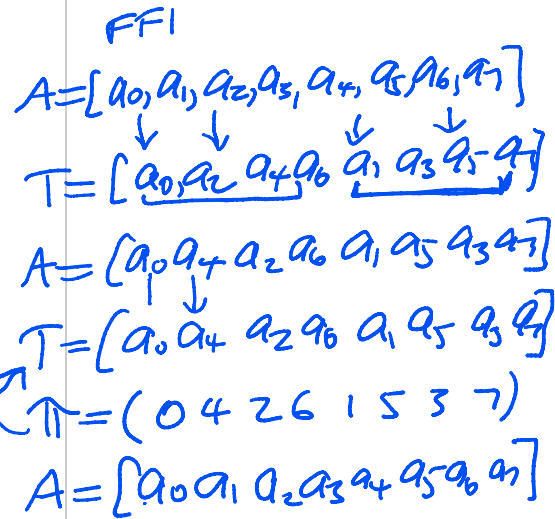
FFT 1 and 2

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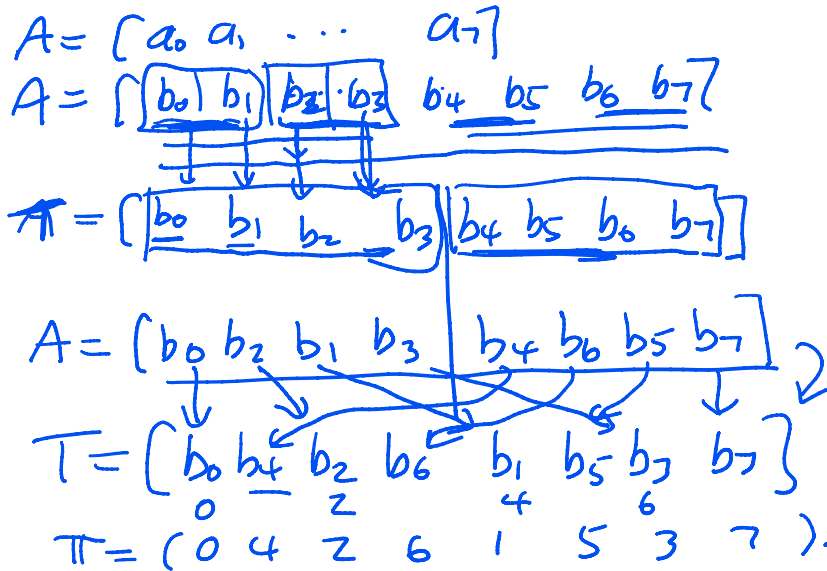
$$W = [1, \omega, \omega^2, \dots, \omega^{n/2-1}, 1, \omega^2, \omega^4, \dots, \omega^{n/2-2}, 1, \omega^4, \omega^8, \dots, \omega^{n/2-4}, \dots, 1, 0]$$

```
void FFT1( int *A, int n,
           int *W, int p, int *T )
{
    int i,n2,t;
    if( n==1 ) return;
    n2 = n/2;
    for( i=0; i<n2; i++ ) {
        T[ i ] = A[2*i];
        T[n2+i] = A[2*i+1];
    }
    FFT1( T, n2, W+n2, p, A );
    FFT1( T+n2, n2, W+n2, p, A+n2 );
    for( i=0; i<n2; i++ ) {
        t = mulmod(W[i],T[n2+i],p);
        A[ i ] = addmod(T[i],t,p);
        A[n2+i] = submod(T[i],t,p);
    }
    return;
}
```

```
void FFT2( int *A, int n,
           int *W, int p, int *T )
{
    int i,n2,t;
    if( n==1 ) return;
    n2 = n/2;
    for( i=0; i<n2; i++ ) {
        T[i] = addmod(A[i],A[n2+i],p);
        t = submod(A[i],A[n2+i],p);
        T[n2+i] = mulmod(t,W[i],p);
    }
    FFT2( T, n2, W+n2, p, A );
    FFT2( T+n2, n2, W+n2, p, A+n2 );
    for( i=0; i<n2; i++ ) {
        A[ 2*i ] = T[i];
        A[2*i+1] = T[n2+i];
    }
    return;
}
```



The two permutations



In-place FFT routines with permutation removed.

```
void FFT1( int *A, int n,
           int *W, int p )
{
    int i,n2,t;
    if( n==1 ) return;
    n2 = n/2;
    FFT1( A,    n2, W+n2, p );
    FFT1( A+n2, n2, W+n2, p );
    for( i=0; i<n2; i++ ) {
        s = A[i];
        t = mulmod(W[i],A[n2+i],p);
        A[ i ] = addmod(s,t,p);
        A[n2+i] = submod(t,t,p);
    }
    return;
}

void FFT2( int *A, int n,
           int *W, int p )
{
    int i,n2,t;
    if( n==1 ) return;
    n2 = n/2;
    for( i=0; i<n2; i++ ) {
        s = addmod(A[i],A[n2+i],p);
        t = submod(A[i],A[n2+i],p);
        A[i] = s;
        A[n2+i] = mulmod(t,W[i],p);
    }
    FFT2( A,    n2, W+n2, p );
    FFT2( A+n2, n2, W+n2, p );
    return;
}
```

$$W = [1, \omega, \omega^2, \dots, \omega^{n/2-1}, 1, \omega^2, \omega^4, \dots, \omega^{n/2-2}, 1, \omega^4, \omega^8, \dots, \omega^{n/2-4}, \dots, 1, 0]$$