A4Q1

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(a) If $\alpha > \beta$ in lex then $\exists k$ s.t. $\alpha_k > \beta_k$ and $\alpha_i = \beta_i$ for $1 \le i < k$.
If we $u = \gamma + \alpha$ and $v = \gamma + \beta$ then $u_k = \alpha_k + \gamma_k > \beta_k + \gamma_k = v_k$
and $u_i = v_i$ for $1 \le i < k$ so $u > v$ in lex order.

(b) $1 > x > x^2 > x^3 > \dots$ has no least monomial so it's not a well ordering so not a monomial ordering.

(c) In $>_{lex}$ with $x>y$ $LT(a) = 10x^3y^2$ and $LT(b) = 2x^2$ so

$$5xy^2 + 7xy + 3y^4 = q$$
$$2x^2+3xy+y^3 \,\big)\, 10x^3y^2 + 14x^3y + 6x^2y^4 + 15x^2y^3 + 21x^2y^2 + 14xy^5 + 7xy^4 + 3y^7$$
$$-(10x^3y^2 + 15x^2y^3 + 5xy^5)$$
$$14x^3y + 6x^2y^4 + 21x^2y^2 + 9xy^5 + 7xy^4 + 3y^7$$
$$-(14x^3y + 21x^2y^2 + 7xy^4)$$
$$6x^2y^4 + 9xy^5 + 3y^7$$
$$-(6x^2y^4 + 9xy^5 + 3y^7)$$
$$0 = r$$

In grlex with $x>y$ $LT(a) = 3y^7$ and $LT(b) = y^3$.

$$3y^4 + 5xy^2 + 7xy$$
$$y^3+2x^2+3xy \,\big)\, 3y^7 + 6x^2y^4 + 14xy^5 + 10x^3y^2 + 15x^2y^3 + 7xy^4 + 14x^3y + 21x^2y^2$$
$$-(3y^7 + 6x^2y^4 + 9xy^5)$$
$$5xy^5 + 10x^3y^2 + 15x^2y^3 + 7xy^4 + 14x^3y + 21x^2y^2$$
$$-(5xy^3 + 10x^3y^2 + 15x^2y^3)$$
$$7xy^4 + 14x^3y + 21x^2y^2$$
$$-(7xy^4 + 14x^3y + 21x^2y^2)$$
$$0$$

(d). In the division algorithm

while $p \neq 0$ do
find i s.t. $LT(f_i) \mid LT(p)$.
CASE 1. if no such i exists $(r, p) \leftarrow r + LT(p),\ p - LT(p)$.
CASE 2. $t \leftarrow LT(p)/LT(f_i)$.

CASE 1. If no such i exists [illegible]

CASE 2. $t \leftarrow LT(p)/LT(f_i)$.

$$(p, a_i) \leftarrow (p - tb, a_i + t).$$

In CASE 1 $\quad p_{new} \leftarrow p_{old} - LT(p_{old})$. So $LT(p_{new}) < LT(p_{old})$.

In CASE 2. $\quad p_{new} \leftarrow p_{old} - t\,b$.

Let us write $p = LT(p) + (p - LT(p))$

this term = those, by prop (ii) of monomial ordering.

$$p - tb = LT(p) + (p - LT(p)) - \frac{LT(p)}{LT(b)}\left(LT(b) + (b - LT(b))\right)$$

$$= LT(p) + (p - LT(p)) - \left[\frac{LT(p)}{LT(b)} LT(b) + \frac{LT(p)}{LT(b)} \cdot (b - LT(b))\right]$$

$$= \cancel{LT(p)} + (p - LT(p)) - \overset{\shortparallel}{\cancel{LT(p)}} + \frac{LT(p)}{LT(b)} \cdot (b - LT(b))$$

$$= \underbrace{p - LT(p)}_{< LT(p)} - \underbrace{\frac{LT(p)}{LT(b)} \cdot (b - LT(b))}_{< LT(p)}$$

Thus $LT(p_{new}) < LT(p_{old})$.