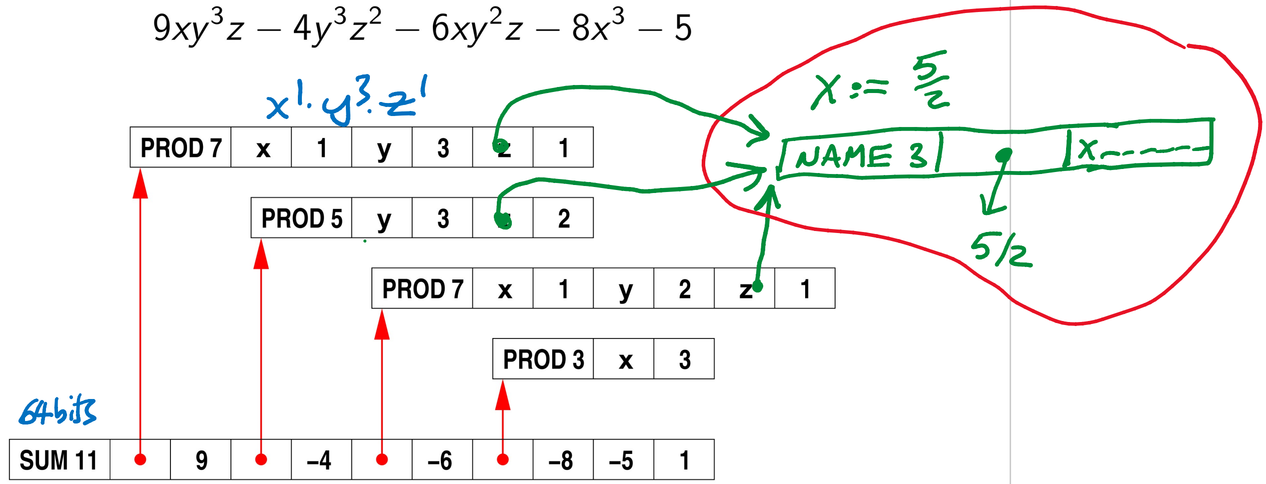
Some Data Structures

October 19, 2023 8:30 AM

Maple's sum of products representation

$$9xy^3z - 4y^3z^2 - 6xy^2z - 8x^3 - 5$$



Space t terms n variables $\leq t(2n+1+2)$

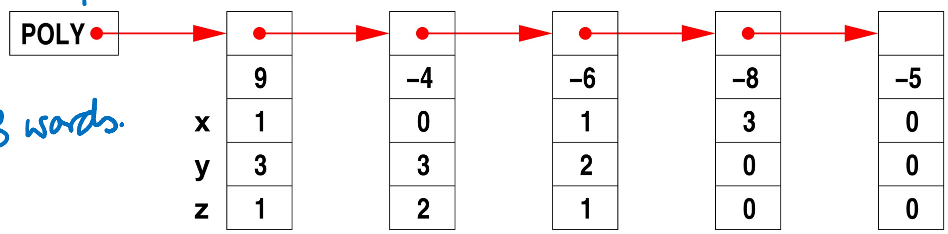
33 words. Monomial multiplication is very slow.

Singular's linked lists representation

$$9xy^3z - 4y^3z^2 - 6xy^2z - 8x^3 - 5$$

R[x].

3 words.



$5 \cdot 5 + 3 = 28$ words

Monomial multiplication is slow, not as slow as Maple.

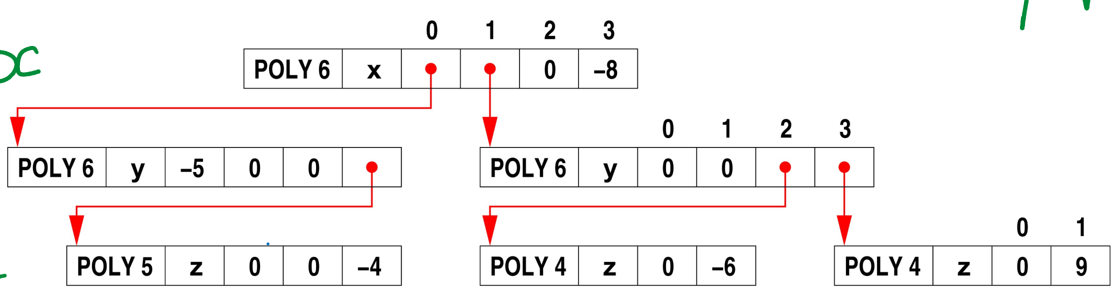
Pari's recursive dense representation

$$(-5 - 4z^2y^3) + (-6zy^2 + 9zy^3)x - 8x^3 \in \mathbb{Z}[z][y][x]$$

x

y

z



Monomial multiplication is fast — add two integers.

Maple's packed monomial arrays

$$9xy^3z - 4y^3z^2 - 6xy^2z - 8x^3 - 5$$

$x^1y^3z^1$

SEQ 4	x	y	z

64bits

POLY 12	•	5131	9	5032	−4	4121	−6	3300	−8	0000	−5

12 words.

Monomials are encoded in 64 bit integers.

$$x^1y^3z^1 = \boxed{5131} = 5\cdot 2^{48} + 1\cdot 2^{32} + 3\cdot 2^{16} + 1\cdot$$

total deg

64 bits

16 16 16 bits 16 bits

Maple's packed monomial arrays cont.

```
> f := 9*x*y^3*z-4*y^3*z^2-6*x*y^2*z-8*x^3-5;
                                   3        3  2          2        3
              f := 9 x y  z - 4 y  z  - 6 x y  z - 8 x  - 5

> dismantle(f);
POLY(12)
   EXPSEQ(4)
      NAME(4): x
      NAME(4): y
      NAME(4): z
   DEGREES(HW): ^5 ^1 ^3 ^1
   INTPOS(2): 9
   DEGREES(HW): ^5 ^0 ^3 ^2
   INTNEG(2): -4
   DEGREES(HW): ^4 ^1 ^2 ^1
   INTNEG(2): -6
   DEGREES(HW): ^3 ^3 ^0 ^0
   INTNEG(2): -8
   DEGREES(HW): ^0 ^0 ^0 ^0
   INTNEG(2): -5
```


POLY DAG (Monagan & Pearce 2012).

Why pack $x^i y^j z^k$ as $\boxed{i+j+k \mid i \mid j \mid k}$ 16 bits each?

- Monomial comparisons are integer comparisons 1 instruction. in grlex order.
- Monomial multiplications are integer additions 1 instruction.

$$(x^2 \cdot y \cdot z^3)(x y^2 z) = x^3 y^3 z^4$$

$$\begin{array}{rl} & \boxed{6 \mid 2 \mid 1 \mid 3} \\ + & \boxed{4 \mid 1 \mid 2 \mid 1} \\ = & \boxed{10 \mid 3 \mid 3 \mid 4} \end{array}$$

- Uses one word of memory for each monomial.
- What if $i+j+k > 2^{16}$?

  We use the SUM of PROD representation!

Why grlex and not lex order?

$n=3$ grlex $\boxed{i+j+k \mid i \mid j \mid k}$ $64/4=16$ bits.

$x^i y^j z^k$ lex $\boxed{\quad i \quad \mid \quad j \quad \mid \quad k \quad}$ $64/3=21$ bits.

lex

$$\begin{array}{r|l} & q = y^2 \\ x+y^2 & xy^2 + y^{3} \\ & -(xy^2 + y^{4}) \\ & -y^4 + y^3 = r \end{array}$$

If 2 bits for each variable y^4 would overflow.

grlex

$$\begin{array}{r|l} & x + y = q. \\ y^2+x & xy^2 + y^3 \\ & -(xy^2 + x^2) \\ & y^3 - x^2 \\ & -(y^3 + xy) \end{array}$$

In grlex degrees can not increase.

$$-\underline{(y^3 + xy)}$$

$$-x^2 - xy = r.$$

If a and b fit in the POLY DAE we can divide a by b in grlex with no overflow but in lex the intermediate monomials may overflow.