MATH 240 Assignment 5, Spring 2016

Please put your name and student ID at the top of the front page and staple your assignment together.
Please hand in to the dropoff boxes outside AQ 4135 by 6pm Tuesday March 29th.
A reminder that quiz 4 is on Wednesday March 30th at the beginning of class.

Michael Monagan

5.1 Exercises 1, 6, 10, 18, 21, 24, 27.

5.2 Exercises 2, 9, 23, 24.

5.3 Exercises 4, 10, 17, 21, 27, 28.

Exercises for Complex numbers – See Appendix B

1. For $x = 2 + i$ and $y = 1 - 3i$ calculate $x + y$, $x - y$, $2x$, xy, x/y, $\overline{x}$, $|y|$ and $\arg x$.

2. Let $x = a + bi$ and $y = c + di$.
   Show that $xy = yx$, $|xy| = |x| \times |y|$ and $\overline{xy} = \overline{x} \times \overline{y}$.

3. Express $x = 2 + i$ in polar co-ordinates, first in the form $|x|(\cos\theta + i\sin\theta)$ then in the form $|x|e^{i\theta}$. Then calculate x^2 in both forms.

4. Use the quadratic formula to solve $x^2 + 2x + 2 = 0$.
   You should get two complex solutions.

5. The polynomial $x^3 - 8$ has one real root $x = 2$ and two complex roots. To find the complex roots first calculate the quotient $(x^3 - 8)/(x - 2)$ using long division.

6. Do exercise 1 and the practice problem in section 5.5.

Exercises for the internet page ranking algorithm

Suppose you have web pages P_1, P_2, P_3 and P_4 with hyperlinks $P_1 \to P_2, P_2 \to P_3, P_3 \to P_1$, $P_3 \to P_4, P_1 \to P_4$ and $P_4 \to P_2$. Assuming a web-surfer takes the hyperlinks on a web page with equal probability, construct the 4 by 4 Markov matrix Q. Now solve $Qq = q$ for the probability vector q such that $q_1 + q_2 + q_3 + q_4 = 1$. Hence determine the ranking of the pages. You will need to row reduce the 4 by 4 matrix $Q - I_4$. If you row reduce $Q - I_4$ to reduced row echelon form you should get

$$\begin{bmatrix} 1 & 0 & 0 & -2/3 \\ 0 & 1 & 0 & -4/3 \\ 0 & 0 & 1 & -4/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercises for the Leslie population distribution model

Suppose we model a population growth using three age groups. Let F_1, F_2, F_3 be the fertility rates and let S_1, S_2, and S_3 be the survival rates of the three age groups. Suppose $F_1 = 0, F_2 = 7/6, F_3 = 7/6, S_1 = 1/2, S_2 = 2/3, S_3 = 2/3$, that is So the Leslie matrix is

$$L = \begin{bmatrix} F_1 & F_2 & F_3 \\ S_1 & 0 & 0 \\ 0 & S_2 & S_3 \end{bmatrix} = \begin{bmatrix} 0 & 7/6 & 7/6 \\ 1/2 & 0 & 0 \\ 0 & 2/3 & 2/3 \end{bmatrix}$$

Calculate the eigenvalues of this matrix L. You should find that one of the eigenvalues is 0 and the other two are simple fractions. If the current population is not zero, is the population growing or declining? What is the long term population distribution vector?

Now S_1, the survival rate of the first age group, is $1/2$. What must S_1 be for the population to stabilize?