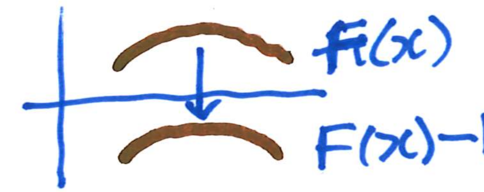
7.5 Strategy for Integration



The Fundamental theorem of Calculus part (2) says

If $f(x)$ is continuous on $[a,b]$ and $F(x)$ is ~~an~~ the antiderivative of $f(x)$

$$\left\{\begin{array}{l} F'(x)=f(x) \ \checkmark \\ F(x)=\int f(x)\,dx \ \checkmark \end{array}\right.$$

then $$\int_a^b f(x)\,dx = F(b)-F(a).$$

We need an antiderivative of $f(x)$.

5.5 Substitution

7.1 $\int f\cdot g' = fg - \int g\cdot f'$

7.2 $\int \sin^m x\cdot\cos^n x\,dx$

7.3 $\sqrt{a^2-x^2}$, $\sqrt{a^2+x^2}$, $\sqrt{x^2-a^2}$

7.4 $\int \frac{3x^2-2}{x^3+3x-4}\,dx$

Final Exam Summer 2004

7.4 $\int \frac{x+4}{x^3+x}\,dx = \int \left(\frac{4}{x} + \frac{-4x+1}{x^2+1}\right)dx = \int \frac{4}{x}\,dx + \int \frac{-4x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$

$u = x^2+1$

$4\ln|x| + -2\ln(1+x^2) + \tan^{-1}x$

7.2 $\int_0^{\pi/2} \sin^4 x \cos^3 x\,dx$

7.1 $\int \underset{f}{e^x} \cdot \underset{g'}{\sin 2x}\,dx$

g', f

$$\frac{x+4}{x^3+x} = \boxed{\frac{x+4}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1} = \frac{4}{x} + \frac{-4x+1}{x^2+1}}$$

$x^2+1=0$

$x^2=-1$

$x=\pm\sqrt{-1}$

$1\cdot x+4 = A(x^2+1) + (Bx+C)x$

x^0	4	$= A\cdot 1$
x^1	1	$= C\cdot 1$
x^2	0	$= A+B \Rightarrow B=-4.$

Fall 2005

$\int \frac{\ln x}{x} dx$

7.2

$\int \cos^2(5x)\,dx$

7.1

$\int \underset{g'}{x^3} \cdot \underset{f}{\ln x}$

7.4

$\int \frac{3x+1}{x(x+1)} dx$

5.5
7.1

$u = \ln x$

$\cos^2 A = \frac{1}{2} + \frac{1}{2}\cos 2A$

$\int \left(\frac{1}{2} + \frac{1}{2}\cos(10x)\right) dx$

$$\int f \cdot g' = f \cdot g - \int g f'$$

$$\int \underset{g'}{\frac{1}{x}} \cdot \underset{f}{\ln x} = \ln x \cdot \underset{g}{\ln x} - \int \ln x \cdot \frac{1}{x}\,dx \Rightarrow 2\int \frac{1}{x}\ln x\,dx = \ln^2 x$$

$$1 \cdot \int \frac{1}{x}\ln x\,dx = \frac{1}{2}\ln^2 x + C.$$

$$\int \frac{1}{x}\ln x\,dx = \int \cancel{\frac{1}{x}} \cdot u \cdot \cancel{x}\,du = \int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}\ln^2 x + C.$$

$u = \ln x \quad \frac{du}{dx} = \frac{1}{x} \Rightarrow dx = x\,du$

Spring 2006

7.1 $\int x^2 \ln^2 x \, dx$

$\cos\pi$

7.2 $\int_0^{\pi} \cos^3 x \boxed{\sin 2x} \, dx = \int_0^{\pi} 2 \cdot \underline{\cos^4 x \cdot \sin^1 x} \, dx = \int_1^{-1} -2u^4 \, du = \cdots$

$\cos 0$

$\sin 2x = 2 \sin x \cos x$

$u = \cos x$

$du/dx = -\sin x$

$dx = -du/\sin x$

7.3 $\int \frac{\sqrt{x2-1}}{x} dx$

7.4 $\int \frac{dx}{x^2 - 3x - 4}$

Summer 200

$\boxed{u = x^3}$

$u = -x^3$

$du/dx = 3x^2 \Rightarrow dx = du/3x^2$

5.5 $\int x^5 e^{-x^3} dx = \int \frac{x^5 \cdot e^{-u} \cdot du}{3x^2} = \int \frac{1}{3} x^3 e^{-u} du = \int \underbrace{\frac{1}{3} u}_{f} \underbrace{e^{-u}}_{g'} du$ 7.1

5.5 $\int_1^5 \sqrt{-x^2+6x-5}\, dx = \int_1^5 \sqrt{4-(x-3)^2}\, du = \int_{-2}^{2} \sqrt{4-u^2}\, du = \frac{1}{2}\pi \cdot 2^2$ 7.3

$u = x-3 \quad du/dx = 1$

$\int \sqrt{\frac{1+x}{1-x}}\, dx = \int \frac{\sin\theta + 1}{\cancel{\cos\theta}} \cdot \cancel{\cos\theta}\, d\theta = \int (1+\sin\theta)\, d\theta$

$= \theta - \cos\theta + C$

$= \sin^{-1}x - \sqrt{1-x^2} + C$

$\sqrt{\frac{(1+x)}{(1-x)} \cdot \frac{(1+x)}{(1+x)}} = \frac{x+1}{\sqrt{1-x^2}}$ 7.3

5.5 $\int \frac{\cos x}{4-\sin^2 x} dx = \int \frac{\cos x}{4-u^2} \frac{du}{\cos x} = \int \frac{du}{4-u^2}$ $4-u^2 = (2-u)(2+u)$

7.4

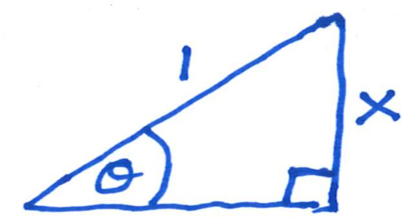
$u = \sin x$

$du/dx = \cos x \quad dx = du/\cos x$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$

$\boxed{x = \sin\theta}$ $\quad dx/d\theta = \cos\theta \Rightarrow dx = \cos\theta\, d\theta.$

$\theta = \sin^{-1}x \quad A^2 + x^2 = 1^2 \Rightarrow A =$



$A = \sqrt{1-x^2}$