MATH 152 Assignment 2, Spring 2024.

Webassign Exercises

5.4 Exercises 5, 15, 32, 59, 71

5.5 Exercises 1, 2, 4, 59

6.1 Exercises 13, 16, 17, 61

6.2 Exercises 5, 11, 19

Written Exercises

1 (Section 5.3)

(a) State the Fundamental Theorem of Calculus part 2.

(b) Let $f(x)$ and $g(x)$ be continuous on $[a, b]$. Apply the Fundamental Theorem of Calculus part 2 to show that

$$\int_a^b (f(x) + g(x))\, dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$$

Hint: let $F(x)$ be an antiderivative of $f(x)$ and $G(x)$ be an antiderivative of $g(x)$.

2 (Section 5.4) Show that $\int \cos(x)^2 dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$ by differentiating both sides and using the trig identities $\sin 2A = 2\sin A \cos A$ and $\cos 2A = 2\cos(A)^2 - 1$.

3 (Section 5.4) Water flows out of a storage tank at a rate of $r(t) = 100 - 10t$ litres per minute. Find the amount of water that flows out of the tank during $0 \leq t \leq 10$.

4 (Section 5.5) Calculate $\int \cot x\, dx$. Use $\cot x = \frac{\cos x}{\sin x}$ and make a substitution.

5 (Section 5.5) Use a substitution to show that $\int_0^4 e^{-\sqrt{x}} dx = \int_0^2 2xe^{-x} dx$.

6 (Section 6.1) Let A be the area between $f(x) = 3 - x^2$ and $g(x) = x^2 - 1$. Sketch A then express A as a definite integral then calculate A using the FTC.

7 Let V be the volume of a cone of height h with a base of radius r. Show that $V = \frac{1}{3}\pi r^2 h$ by expressing V the volume of revolution about the x axis and evaluating the integral that you get.

8 (Section 6.2) Consider two spheres both of radius r which are placed on the x axis with the centre of the first sphere at $x = 0, y = 0$ and the centre of the second sphere at $x = r, y = 0$ so that they intersect each other. Show that the volume in common is $\frac{5}{12}\pi r^3$.
Sketch the two spheres and the volume to be calculated first.

Midterm 1 is on Friday February 2nd in class.

It covers the material covered on Assignments 1 and 2 which is Sections 4.9, 5.1−5.5, 6.1, and 6.2.