

# Logistic Growth with Harvesting

```
> restart;
> with(DEtools):
> LGH := diff( y(t),t) = a*y(t)*(Ym-y(t))-H;
```

$$LGH := \frac{d}{dt} y(t) = a y(t) (Y_m - y(t)) - H \quad (1)$$

```
> Ym := 10; # the carrying capacity
a := 0.01; # natural growth rate = a*Ym = 0.1
H := 0.2; # constant harvesting rate
```

```
Ym:= 10
```

```
a:= 0.01
```

```
H:= 0.2
```

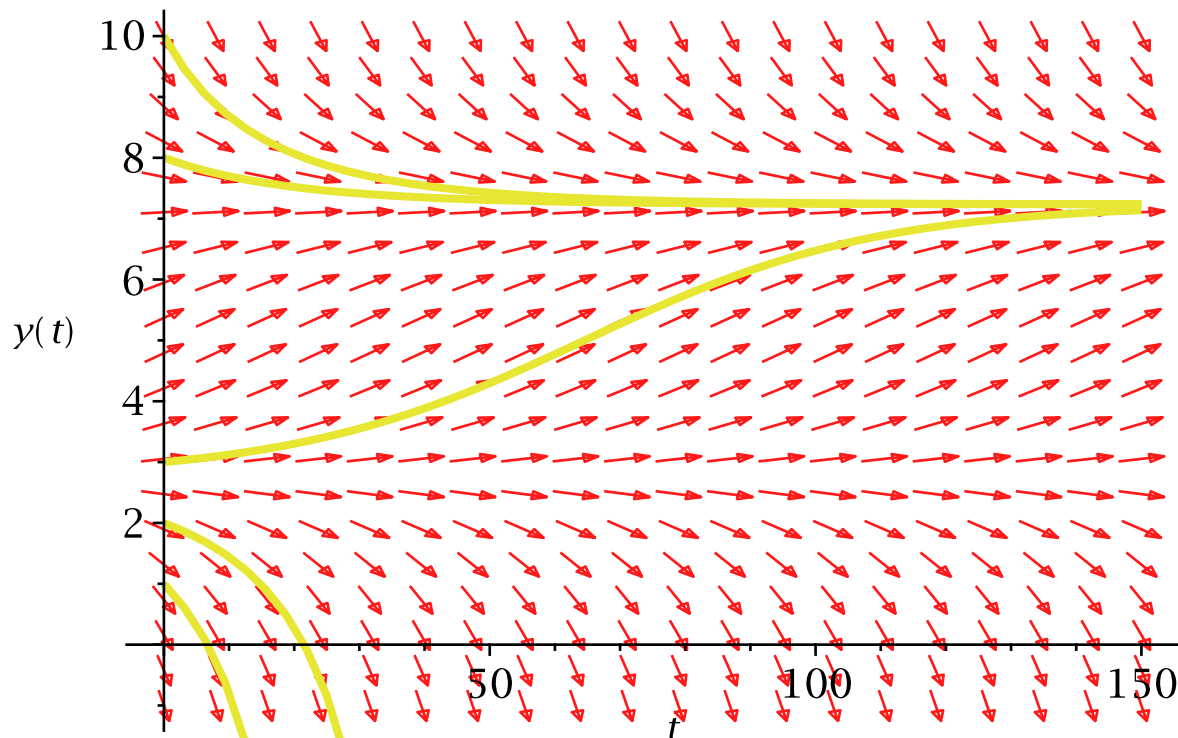
(2)

```
> initvals := [y(0)=1,y(0)=2,y(0)=3,y(0)=8,y(0)=10];
```

```
initvals:= [y(0) = 1, y(0) = 2, y(0) = 3, y(0) = 8, y(0) = 10]
```

(3)

```
> DEplot( LGH, y(t), t=0..150, y=-1..10, initvals, arrows=medium );
```



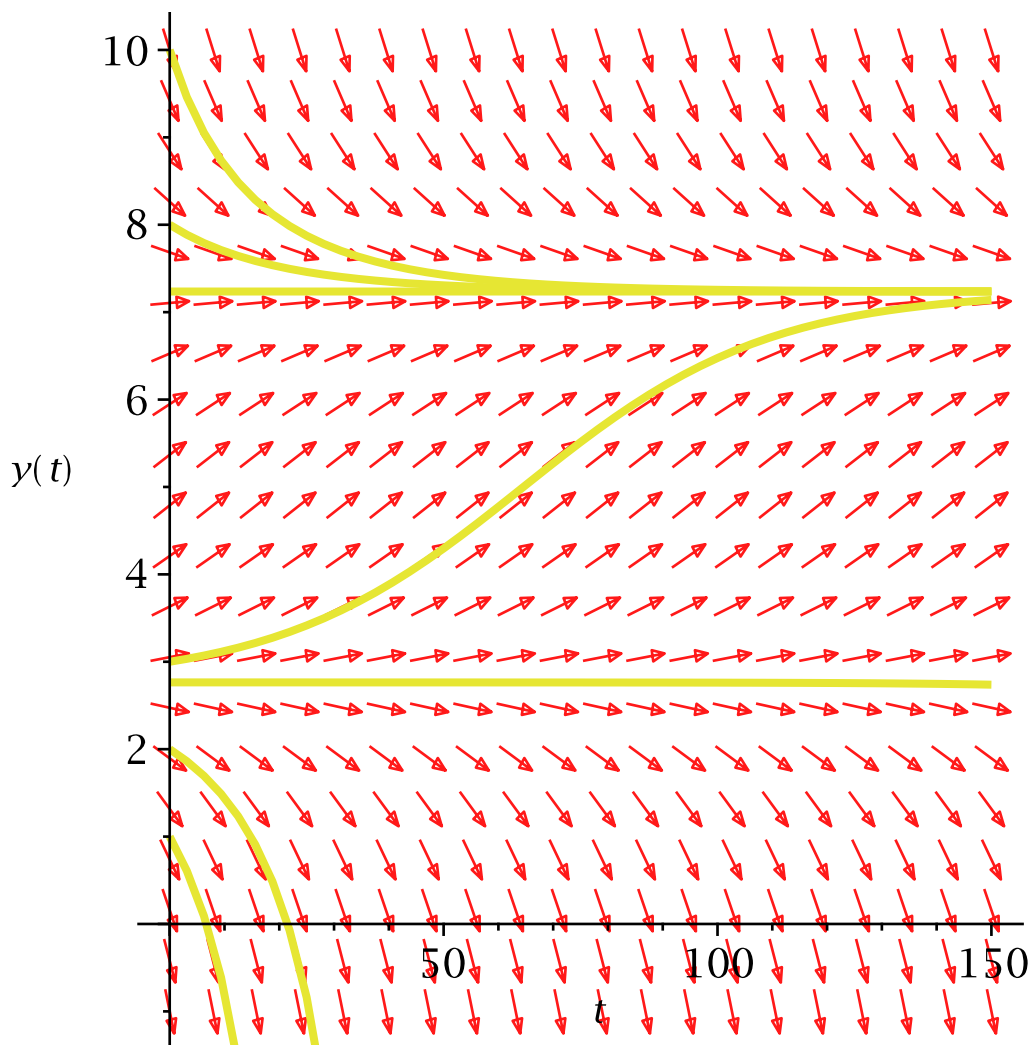
- 1: From the graph, the new maximum carrying capacity  $C$  is less than 8. What is it? Hint: it must be a point between 6 and 8 where  $y' = 0$ .
- 2: From the graph, the population dies out for  $y(0)=2$  but increases for  $y(0)=3$ . Let  $M$  be the smallest value of  $y(0)$  that supports this harvest rate. What is it?
- 3: Verify 1 and 2 by including  $y(0)=C$  and  $y(0)=M$  in the plot.
- 4: Try larger values of  $H$  to determine the maximum sustainable harvest rate  $H_{max}$ , that is, for  $H < H_{max}$  there is a  $y(0)$  for which the population does not die out and for  $H > H_{max}$  the population dies out for all  $y(0)$ .
- 5: The maximum sustainable harvest rate  $H_{max}$  occurs when  $C = M$ . Can you determine a formula for  $H_{max}$  in terms of  $a$  and  $Y_m$ .

## Solution

```
> solve( rhs(LGH), y(t) );
2.763932022, 7.236067978 (1.1)
```

```
> initvals := [op(initvals), y(0)=2.7639, y(0)=7.2361];
initvals:= [y(0) = 1, y(0) = 2, y(0) = 3, y(0) = 8, y(0) = 10, y(0) = 2.7639,
y(0) = 7.2361] (1.2)
```

```
> DEplot( LGH, y(t), t=0..150, y=-1..10, initvals, arrows=medium
);
```



```
> H := 0.25;
H:= 0.25 (1.3)
```

```
> solve( rhs(LGH), y(t) );
5., 5. (1.4)
```

```
> initvals := [y(0)=10,y(0)=7,y(0)=5,y(0)=4.5,y(0)=4];
initvals:= [y(0) = 10, y(0) = 7, y(0) = 5, y(0) = 4.5, y(0) = 4] (1.5)
```

```
> DEplot( LGH, y(t), t=0..200, y=-1..10, initvals, arrows=medium
);
```

