Critical Points and calculating e^x .

Michael Monagan, October 9, 2018

```
> f := 1-x^2-y^3-2*x*y;
```
$$f := -y^3 - x^2 - 2xy + 1 \tag{1}$$

```
> fx := diff(f,x);
```
$$fx := -2x - 2y \tag{2}$$

```
> fy := diff(f,y);
```
$$fy := -3y^2 - 2x \tag{3}$$

```
> sys := {fx=0,fy=0};
```
$$sys := \{-2x - 2y = 0, \ -3y^2 - 2x = 0\} \tag{4}$$

```
> crit := solve(sys);
```
$$crit := \{x = 0, y = 0\}, \ \left\{x = -\frac{2}{3}, y = \frac{2}{3}\right\} \tag{5}$$

```
> fxx := diff(f,x,x);
```
$$fxx := -2 \tag{6}$$

```
> fyy := diff(fy,y);
```
$$fyy := -6y \tag{7}$$

```
> fxy := diff(fx,y);
```
$$fxy := -2 \tag{8}$$

```
> local D;
```
Warning, A new binding for the name `D` has been created. The global instance of this name is still accessible using the :- prefix, :-`D`. See ?protect for details.

```
> D := fxx*fyy-fxy^2;
```
$$D := 12y - 4 \tag{9}$$

```
> eval([fxx,fyy,D],crit[1]);
```
$$[-2, 0, -4] \tag{10}$$

So D < 0 which means we have a saddle

```
> eval([fxx,fyy,D],crit[2]);
```
$$[-2, -4, 4] \tag{11}$$

So D > 0 and fxx < 0 and fyy < 0 which means we have a local maximum. Let's check
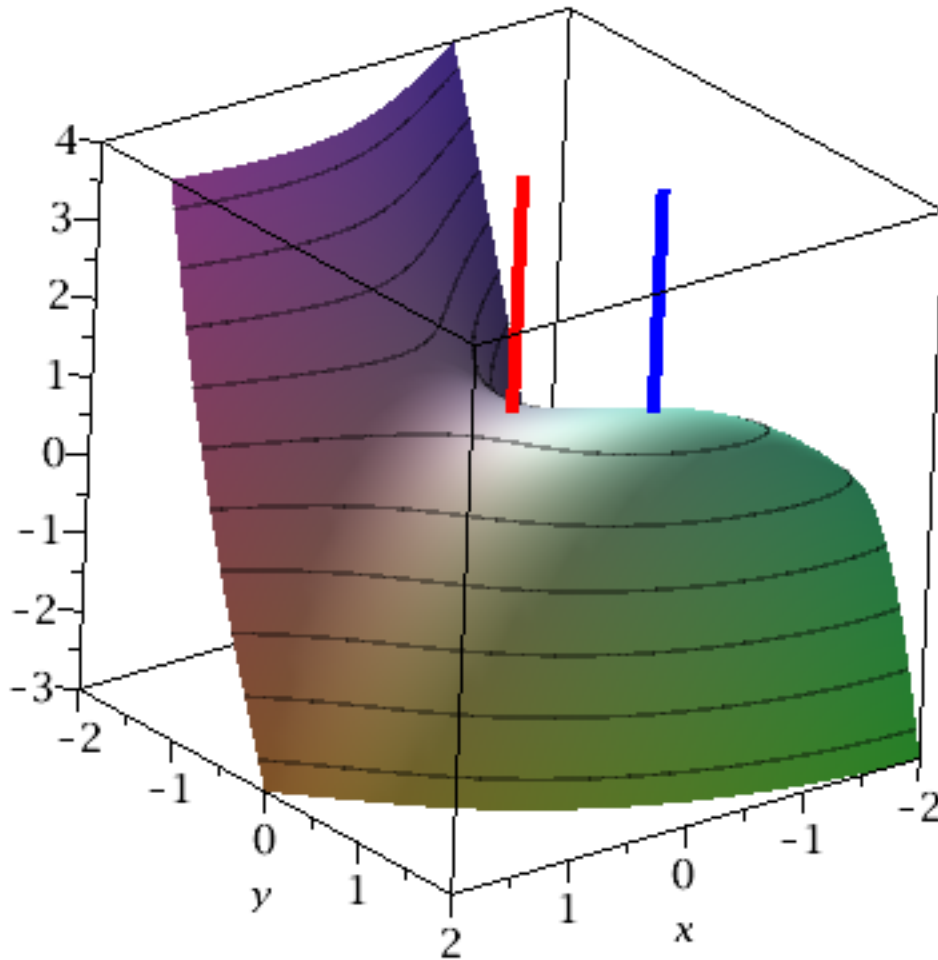
```
> with(plots):
> s1 := spacecurve( [0,0,z], z=-5..5, color=red, thickness=5 ):
  s2 := spacecurve( [-2/3,2/3,z], z=-5..5, color=blue, thickness=5
  ):
  fp := plot3d( f, x=-2..2, y=-2..2, style=patchcontour ):
> display( [s1,s2,fp], view=[default,default,-3..4] );
```

Calculating exp(x)

```
> T := taylor(exp(x),x=0,8);
```

$$T := 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \frac{1}{720} x^6 + \frac{1}{5040} x^7 + O(x^8)$$ **(12)**

```
> P := convert(T,polynom);
```

$$P := 1 + x + \frac{1}{2} x^2 + \frac{1}{6} x^3 + \frac{1}{24} x^4 + \frac{1}{120} x^5 + \frac{1}{720} x^6 + \frac{1}{5040} x^7$$ **(13)**

```
> numer(P);
```

$$x^7 + 7 x^6 + 42 x^5 + 210 x^4 + 840 x^3 + 2520 x^2 + 5040 x + 5040$$ **(14)**

```
> EXP := proc(x) (x^7+7*x^6+42*x^5+210*x^4+840*x^3+2520*x^2+5040*
  x+5040)/5040 end;
```

$EXP := \mathbf{proc}(x)$ **(15)**

$1 + x + 1/2*x^2 + 1/6*x^3 + 1/24*x^4 + 1/120*x^5 + 1/720*x^6 + 1$
$/5040*x^7$

end proc

```
> EXP(0.1);
```

$$1.105170918$$ **(16)**

```
> exp(0.1);
```
$$1.105170918 \qquad (17)$$

```
> EXP(3.0);
```
$$19.84642857 \qquad (18)$$

```
> exp(3.0);
```
$$20.08553692 \qquad (19)$$

```
> EXP := proc(y::numeric) local x;
    x := evalf(y);
    if x<0 then 1/EXP(-x);
    elif x>0.1 then EXP(x/2)^2;
     else (x^7+7*x^6+42*x^5+210*x^4+840*x^3+2520*x^2+5040*x+5040)
  /5040
     fi;
  end:
> EXP(3);
```
$$20.08553687 \qquad (20)$$

```
> EXP(3.0)-exp(3.0);
```
$$-5.10^{-8} \qquad (21)$$