MACM 204 Makeup Assignment - Practice Final.

Fall 2014

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You do not have to do this assignment.
It is meant to be a practice assignment, a warm up for the final exam.
However, if you got a bad mark on one of your assignments, or you missed an assignment, you may do this assignment and I'll replace your worst mark with your mark for this one.

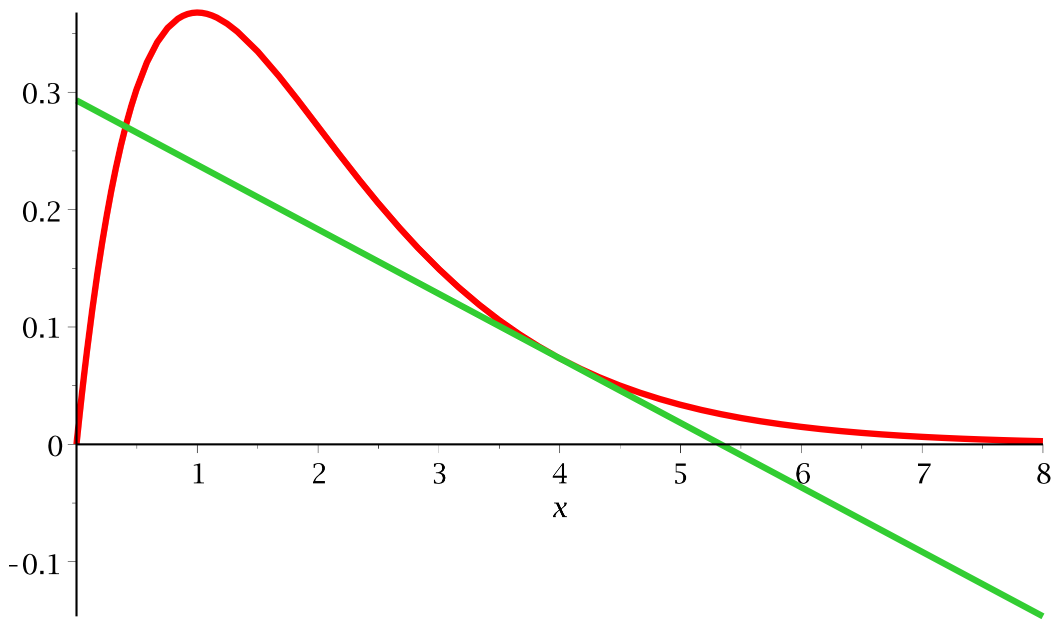
Due Monday December 8th at 5pm.

There are 8 questions. There are 90 marks.
I will mark it out of 80. So you don't need to do all questions to get full marks.
But if you do all questions and you get X marks out of 90 I will give you max(X,90).
If you've forgotten how to use a command, just look at the examples in the help page for that command. If you get stuck, please feel free to come and ask me for help. My office phone number is (778) 782 4279.

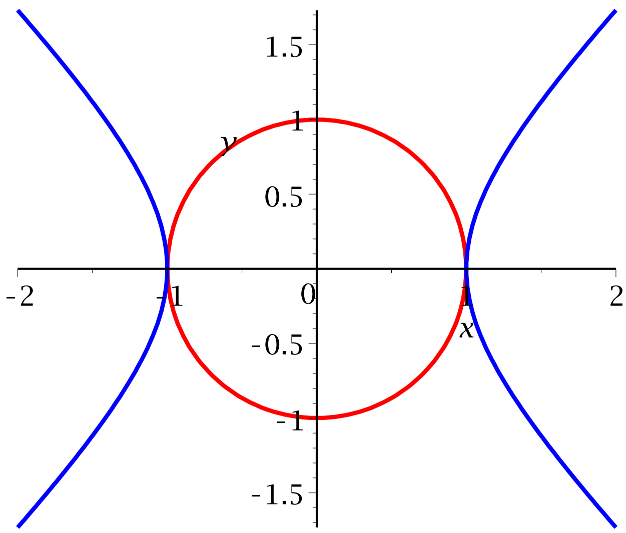
Please attempt each question in a seperate worksheet.
Print your Maple worksheets (double sided if you wish) and hand them in to me.

Question 1 (10 marks)

Part a) Shown below is a plot of $f(x) = x \cdot \exp(-x)$ and the tangent line $T(x)$ at $x = 4$.
Recreate this plot.



Part b) Shown in the plot below is a plot of $x^2 - y^2 = 1$ and $x^2 + y^2 = 1$.
Recreate the plot using the **implicitplot** command. Please make it "smooth".

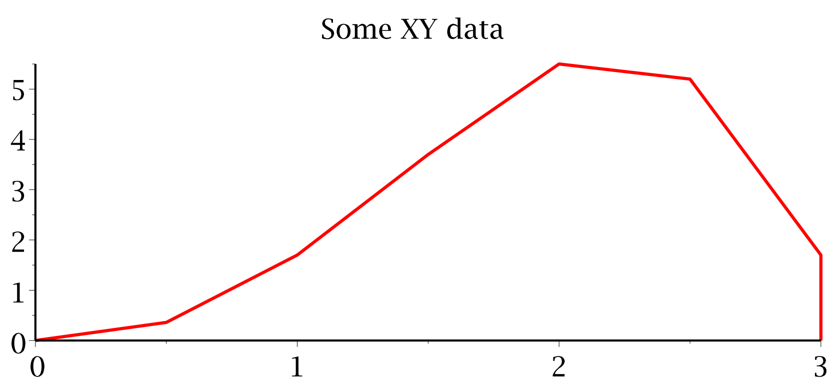


Question 2 (10 marks)

```
> Data := [[0.0,0.0], [0.50,0.36], [1.,1.7], [1.5,3.7],
          [2.0,5.5], [2.5,5.2], [3.0,1.7], [3.0,0.0]];
```

Data := [[0., 0.], [0.50, 0.36], [1., 1.7], [1.5, 3.7], [2.0, 5.5], [2.5, 5.2], [3.0, 1.7], [3.0, 0.]]

Below is a plot of the data points with lines between them.



First, recreate the plot of the data using the **plot** command.
Second, write a Maple procedure **TrapezoidalRulePlot(Data)** that on input of a list of data points, each point of the form $[x_i, y_i]$, computes the area of the trapezoids to approximate the area under the solid curve in the figure above. Test your Maple procedure on the data above. You should get 8.655.

Question 3 (12 marks)

Consider the function $f(x, y)$ below. Compute the formula for the tangent plane $T(x, y)$ a $x=1, y=1$. Generate a plot of $f(x, y)$ AND the tangent plane on the same plot using the **plot3d** command. Choose options for the tangent plane and $f(x, y)$ so that each is distinct and easily visible.

```
> f := 1-x^2-y^2-2*x;
```

$$f := -x^2 - y^2 - 2x + 1$$

Now on a second plot, graph the surface $f(x, y)$ and the curves $f(1, y)$ and $f(x, 1)$ on the same plot.
Use the **spacecurve** command from the **plots** package for the curves. Use $0 \le x \le 2$ and $0 \le y \le 2$ for the domain.

Question 4 (12 marks)

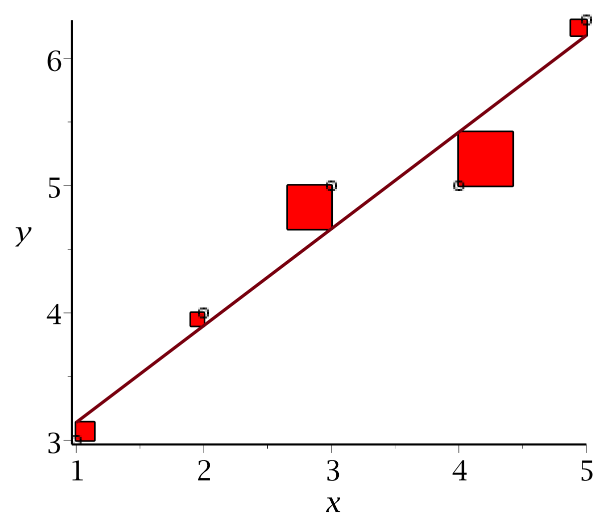
Suppose we have some [x,y] data points

```
> Data := [[1,3],[2,4],[3,5],[4,5],[5,6.3]];
```

$$Data := [[1, 3], [2, 4], [3, 5], [4, 5], [5, 6.3]]$$

Suppose we approximate the data with best line using least squares

```
> with(Student):
> with(LinearAlgebra):
> infolevel[Student[LinearAlgebra]] := 2:
> LeastSquaresPlot( Data, [x,y], curve=a+b*x, boxoptions=[color=red] )
  ;
Fitting curve: 2.380+.7600*x
Least squares error: .5797
Maximum error: .4200
```



Least-squares fit of the curve $y = b\,x + a$ to 5 given data points.

So according to the output, the line of least squares fit is $y = f(x)$ where

$f(x) = 2.38 + 0.76 \cdot x$.
Create the plot by graphing the line $2.38 + 0.76 \cdot x$ then creating red squares using POLYGONS(...) or the **rectangle** command in the plottools package. Don't worry about the points. So the third square goes from [3, $f(3)$] up to the first data point [3,5]. I suggest you write a little procedure **MakeSquare**(*A, B*) that and takes as input the two points $A = [x, f(x)]$ and $B = [x, y]$ and outputs one square.

Question 5 (12 marks)

Consider linear systems of the following form.
Let *A* be an *n* by *n* matrix with 2 on the diagonal and 1 on the super diagonal and 1 on the sub diagonal and 0 else where. Let *b* is a vector of 1's and let *x* be the solution of the linear system $A\,x = b$.
For example, for *n*=6 we have

$$A := \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \quad b := \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad x := \begin{bmatrix} \frac{3}{7} \\ \frac{1}{7} \\ \frac{2}{7} \\ \frac{2}{7} \\ \frac{1}{7} \\ \frac{3}{7} \end{bmatrix}$$

Now solve the linear systems for *n*=4,5,6,7, ... and determine a formula for the solutions. Use the **LinearSolve** command in the **LinearAlgebra** package. You will find that the formula depends on whether *n* is even or odd.

Question 6 (12 marks)

a) factor the polynomial $1 - x^6$

b) evaluate and simplify the sum $\sum_{i=0}^{n} i^2 \cdot 2^i$

c) solve the linear system $\{\alpha \cdot A - \beta \cdot A \cdot B = 1, \beta \cdot A \cdot B - \alpha \cdot B = 0\}$ for A and B using solve

d) evaluate and simplify $\int_{\alpha}^{\beta} (x - \alpha) \cdot (x - \beta) \, dx$

e) solve the DE $y'(t) = k \cdot y(t) - H$ using dsolve with initial value $y(0) = 0$.

f) Consider the function $f(x) = x^3 - 5 \cdot x^2 - 5$.
Using the seq command construct the sequence $f(1), f(2), f(3), \ldots, f(9)$.

Question 7 (10 marks)

For $n = 1, 2, 3, \ldots$ let $P(n)$ be the number of primes $\leq n$.
So that $P(1) = 0$, $P(2) = 1$, $P(3) = 2$, $P(4) = 2$, $P(5) = 3$, etc.

Write Maple procedure **PRIMES** that returns a Maple Array *P* of length *n* such that
$P[1] = 0$, $P[2] = 1$, $P[3] = 2$, $P[4] = 2$, $P[5] = 3$ etc.
Your procedure should intialize **P := Array(1..n);**
Use the **isprime** command to test for primality.
Test your procedure on PRIME(30)

Question 8 (12 marks)

Consider the function $f(x) = \dfrac{\sin(x)}{x}$.

a) Graph $y = f(x)$ and the line $y = x$ on the same plot on the domain $0 \leq x \leq 2$.
b) Solve $f(x) = x$ to 5 decimal places.
c) Calculate $f'(1)$ to 5 decimal places.
d) Evaluate $\int_0^1 f(x)\,\mathrm{d}x$ to 5 decimal places.
e) Expand $f(x)$ as a Taylor series to $\mathrm{O}(x^{10})$.
f) Calculate the $\lim_{x \to 0} f(x)$ in Maple.