

The Lead model

A(t) is the amount of lead in the blood at time t

B(t) is the amount of lead in the bones at time t

J is the amount of lead going into the body from the environment (assume constant)

```
> eq1 := diff(A(t),t) = J - k1*A(t) - k3*A(t) + k2*B(t);
```

```
eq2 := diff(B(t),t) = k1*A(t) - k2*B(t);
```

$$eq1 := \frac{d}{dt} A(t) = J - k1 A(t) - k3 A(t) + k2 B(t)$$

$$eq2 := \frac{d}{dt} B(t) = k1 A(t) - k2 B(t) \quad (1)$$

```
> Equilibrium := { rhs(eq1) = 0, rhs(eq2) = 0 };
```

```
Equilibrium := { k1 A(t) - k2 B(t) = 0, J - k1 A(t) - k3 A(t) + k2 B(t) = 0 } \quad (2)
```

```
> solve( Equilibrium, {A(t),B(t)} );
```

$$\left\{ A(t) = \frac{J}{k3}, B(t) = \frac{k1 J}{k2 k3} \right\} \quad (3)$$

```
> sol := dsolve( {eq1,A(0)=0,eq2,B(0)=0}, {A(t),B(t)} );
```

UGLY FORMULA note shown

```
> J := 1;
```

```
k1 := 0.1;
```

```
k2 := 0.02;
```

```
k3 := 0.1;
```

$$J := 1$$

$$k1 := 0.1$$

$$k2 := 0.02$$

$$k3 := 0.1 \quad (4)$$

```
> sol;
```

```
{A(t) = -5.497518582 e-0.0095012438 t - 4.502481402 e-0.2104987562 t + 10.00000000, \quad (5)
```

```
B(t) = -52.36352270 e-0.009501243800 t + 2.363522729 e-0.2104987562 t
```

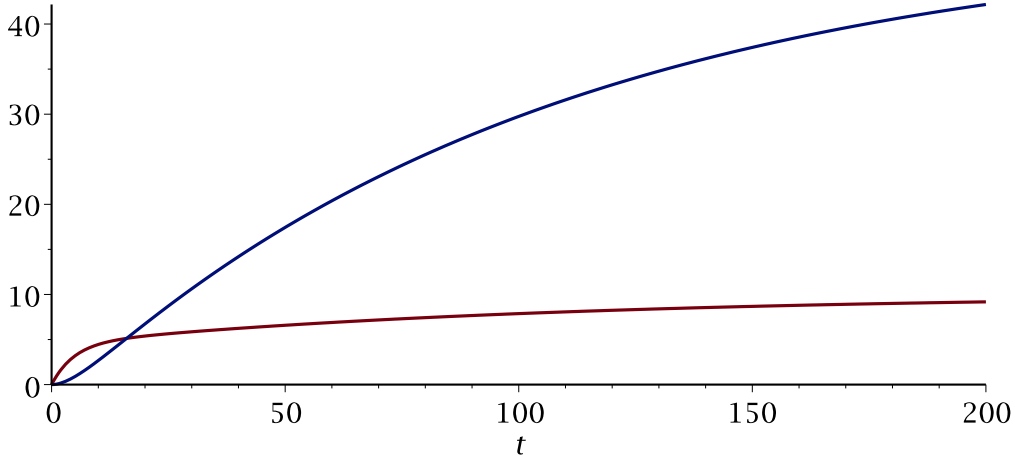
```
+ 50.00000000}
```

```
> map( rhs, sol );
```

```
{-5.497518582 e-0.0095012438 t - 4.502481402 e-0.2104987562 t + 10.00000000, \quad (6)
```

```
-52.36352270 e-0.009501243800 t + 2.363522729 e-0.2104987562 t + 50.00000000}
```

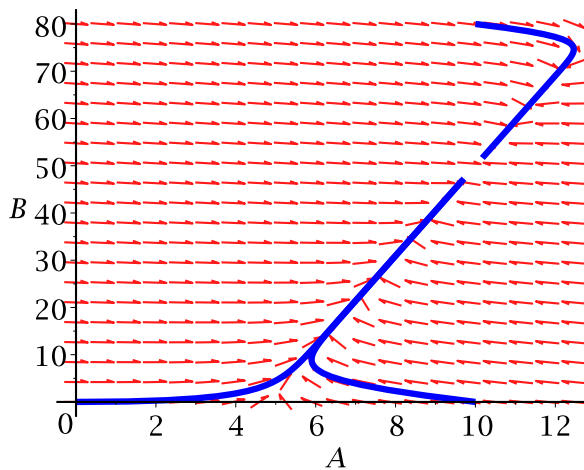
```
> plot( map(rhs,sol), t=0..200 );
```



```
> solve( Equilibrium, {A(t),B(t)} );
      {A(t) = 10., B(t) = 50.}
```

(7)

```
> with(DEtools):
> DEplot( {eq1,eq2}, {A(t),B(t)}, t=0..300,
  {[A(0)=0,B(0)=0],[A(0)=10,B(0)=0],[A(0)=10,B(0)=80]},
  linecolor=blue, numpoints=200 );
```



```
> restart;
```

The House heating model

We have a house with two rooms A and B and B has a furnace F in it
 A(t) is the temperature in room A and
 B(t) is the temperature in room B and
 Am is the outside air temperature.

```
> de1 := diff(A(t),t) = +k1*(B(t)-A(t))-k2*(A(t)-Am);
      de1:=  $\frac{d}{dt} A(t) = k1 (B(t) - A(t)) - k2 (A(t) - Am)$ 
```

(8)

$$\begin{aligned} > \text{de2} := \text{diff}(B(t),t) = -k1*(B(t)-A(t))-k3*(B(t)-Am)+F; \\ \text{de2} := \frac{d}{dt} B(t) = -k1 (B(t) - A(t)) - k3 (B(t) - Am) + F \end{aligned} \quad (9)$$

$$\begin{aligned} > \text{TempEquil} := \text{solve}(\text{map}(\text{rhs}, \{\text{de1}, \text{de2}\}), \{A(t), B(t)\}); \\ \text{TempEquil} := \left\{ A(t) = \frac{k1 k3 Am + k1 F + k3 k2 Am + k1 k2 Am}{k3 k1 + k3 k2 + k1 k2}, B(t) \right. \\ \left. = \frac{k1 k3 Am + k1 F + k3 k2 Am + k1 k2 Am + k2 F}{k3 k1 + k3 k2 + k1 k2} \right\} \end{aligned} \quad (10)$$

$$\begin{aligned} > \text{collect}(\text{TempEquil}, F); \\ \left\{ A(t) = \frac{k1 F}{k3 k1 + k3 k2 + k1 k2} + \frac{k1 k3 Am + k3 k2 Am + k1 k2 Am}{k3 k1 + k3 k2 + k1 k2}, B(t) \right. \\ \left. = \frac{(k1 + k2) F}{k3 k1 + k3 k2 + k1 k2} + \frac{k1 k3 Am + k3 k2 Am + k1 k2 Am}{k3 k1 + k3 k2 + k1 k2} \right\} \end{aligned} \quad (11)$$

$$\begin{aligned} > \text{collect}(\text{TempEquil}, F, \text{simplify}); \\ \left\{ A(t) = \frac{k1 F}{k3 k1 + k3 k2 + k1 k2} + Am, B(t) = \frac{(k1 + k2) F}{k3 k1 + k3 k2 + k1 k2} + Am \right\} \end{aligned} \quad (12)$$

$$> \text{dsolve}(\{\text{de1}, \text{de2}, A(0)=Am, B(0)=Am\}, \{A(t), B(t)\});$$

Ugly

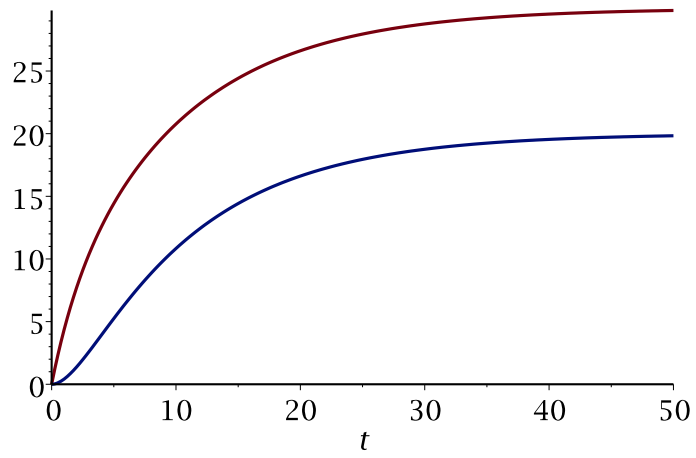
$$\begin{aligned} > Am := 0; \\ k2 := k3; \\ k1 := 2*k2; \\ Am := 0 \\ k2 := k3 \\ k1 := 2 k3 \end{aligned} \quad (13)$$

$$\begin{aligned} > \text{dsolve}(\{\text{de1}, \text{de2}, A(0)=Am, B(0)=Am\}, \{A(t), B(t)\}); \\ \left\{ A(t) = \frac{1}{5} \frac{-\frac{5}{2} e^{-tk3} F + \frac{1}{2} e^{-5tk3} F + 2 F}{k3}, B(t) = \frac{1}{5} \frac{-\frac{5}{2} e^{-tk3} F - \frac{1}{2} e^{-5tk3} F + 3 F}{k3} \right\} \end{aligned} \quad (14)$$

$$\begin{aligned} > F := 5; \\ k3 := 0.1; \\ F := 5 \\ k3 := 0.1 \end{aligned} \quad (15)$$

$$\begin{aligned} > \text{sol} := \text{dsolve}(\{\text{de1}, \text{de2}, A(0)=Am, B(0)=Am\}, \{A(t), B(t)\}); \\ \text{sol} := \left\{ A(t) = 5 e^{-\frac{1}{2} t} - 25 e^{-\frac{1}{10} t} + 20, B(t) = -5 e^{-\frac{1}{2} t} - 25 e^{-\frac{1}{10} t} + 30 \right\} \end{aligned} \quad (16)$$

$$> \text{plot}(\text{map}(\text{rhs}, \text{sol}), t=0..50);$$



```
> with(DEtools):
DEplot( {de1,de2}, {A(t),B(t)}, t=0..40, A=0..50,B=0..50,
[[A(0)=0,B(0)=0],[A(0)=50,B(0)=30]], linecolor=blue );
```

