Assignment 3, MACM 204, Fall 2013

Due Tuesday October 22nd at 4:30pm at the beginning of class.
Late penalty: -20% for up to 48 hours late. 0 after that.

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Please attempt each question in a seperate worksheet (so that you don't destroy your previous work). Print your Maple worksheets (you may print double sided if you wish) and hand them in to me.

There are 8 questions.

Question 1

The easiest way to get a plot of a circle of radius r is to graph the equation of the circle $x^2 + y^2 = r^2$ using the implicitplot command. But this is expensive. A much faster (and more accurate) way is to use the trigonometric parameterization of the circle which is

$x(t) = r \cdot \cos(t),\ y(t) = r \cdot \sin(t)$ for $-\pi \le t \le \pi$.

You can get a parametric plot in Maple using the plot command of $x(t)$ and $y(t)$ for $a \le t \le b$ as follows

```
plot( [x(t),y(t),t=a..b] );
```

Graph the circle for radius 1 using the style=point option so you can see that the points computed are equally spaced around the circle.

Now the implicit equation for a sphere of radius r is $x^2 + y^2 + z^2 = r^2$. Find a trigonometric parameterization for the sphere of radius r, e.g., using google. It will be of the form

$(\ x(s, t),\ y(s, t),\ z(s, t)\ )$ for $a \le s \le b$ and $c \le t \le d$

i.e. there are two parameters s and t.
Graph it for radius 2 using the plot3d command as follows

```
plot3d( [x(s,t), y(s,t), z(s,t)], s=a..b, t=c..d );
```

Try also the style=points option.

Solution 1

Question 2

Solve the following linear systems in Maple using the solve command.

```
> sys1 := [x+y+z=2, x-y=1, x+z=0];
```

$$[x + y + z = 2,\ x - y = 1,\ x + z = 0]$$

```
> sys2 := [x+y+z=2, x-y=1, 2*y+z=1];
```

$$[x + y + z = 2,\ x - y = 1,\ 2\,y + z = 1]$$

```
> sys3 := [x+y+z=2, x-y=1, 2*y+z=-1];
```

$$[x + y + z = 2,\ x - y = 1,\ 2\,y + z = -1]$$

Recall that an equation of the form $a \cdot x + b \cdot y + c \cdot z = d$ is the equation of a plane in 3 dimensions. Explain Maple's answers visually by graphing each system (of three planes) on the same plot using the implicitplot3d command for a suitable domain. Give the three planes different colours. A one sentence explanation is sufficient.

Solution 2

Question 3

Consider the function

```
> f := 4*y^3+x^2-12*y^2-36*y+x^3+2;
```

$$4y^3+x^2-12y^2-36y+x^3+2$$

Find the critical points using Maple. You should get 4 of them. Determine which are saddle points, which are local minimums and which are local maximums.

Graph the surface $f(x, y)$ and highlight the critical points by drawing a spike (vertical line) through them using the **spacecurve** command in the plots package. Graph the saddles in a different color than the others so we can tell which is which.

Solution 3

Question 4

Without exectuting the following Maple program, explain in one sentence, what it does.

```
g := proc(M::list,n::integer) local f;
     if n=0 then return 0 fi;
     if M[1]<50 then f := 1; else f := 0; fi;
     f + g(M,n-1);
  end;
```

Solution 4

Question 5

Consider a function $f(x)$ and the area $A = \int_a^b f(x)\,\mathrm{d}x$. One way to approximate this integral is to use the Trapezoidal rule. Recall that the Trapezoidal rule Tn on n intervals of equal size $h = (b-a)/n$ is given by

$$T_n = \frac{h}{2}\cdot[\ f(a) + 2\cdot f(a+h) + 2\cdot f(a+2\cdot h) + \ldots 2\cdot f(a+(n-1)\cdot h) + f(b)\ ].$$

Write a Maple procedure `Trapezoidal( f, a, b, n )` that calculates Tn. Here the input f should be a Maple function. Use your Trapezoidal procedure as shown below to

estimate

$\int_0^{\frac{\pi}{4}} \sin(x)\,\mathrm{d}x$. You should get the values I have below. Compute also $\int_0^{\frac{\pi}{4}} \sin(x)\,\mathrm{d}x$ to 10 decimal places in Maple using Maple's **int** command.

```
> f := x -> sin(x);
```

$$f := x \rightarrow \sin(x)$$

```
> Trapezoidal( f, 0, Pi/4, 4 );
```

0.2919516176

```
> Trapezoidal( f, 0, Pi/4, 8 );
```

0.2926579321

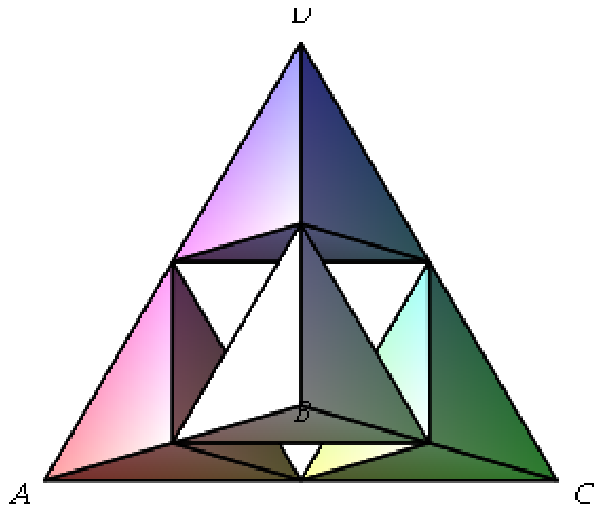
Now estimate $\int_0^{\frac{\pi}{4}} \sqrt{x - \sin(x)}\,\mathrm{d}x$ to 3 decimal places using your Trapezoidal procedure. You will need to define $f(x) = \sqrt{x - \sin(x)}$ as a function. Try to compute the antiderivative $\int \sqrt{x - \sin(x)}\,\mathrm{d}x$ in Maple. What happens?

Solution 5

Question 6

Consider a tetrahedron with vertices A,B,C,D where A,B,C is the base and D is the top. To generate Serpinski's gasket in 3D we need to compute the midpoints of each side (there are 6 sides) and form 4 tetrahedrons from the corners of A,B,C,D with sides half the length as shown in the following figure.

```
> Serpinski3d(1);
```



If we repeat this for each of the four tetrahedra we will have 16 tetrahedra. Repeating this n times will give us 4^n tetrahedra. Program this construction recursively so that Serpinski3d(n) outputs a Maple 3D plot of the the 4^n tetrahedra for n=4 say. Your program should output a PLOT3D(bunch of polygons , AXESSTYLE(NONE)). Hand in a printout for your procedure for $n = 3$.

Now rotate the plot so that the 4^n tetrahedra **cover the plane**. That is, so that there are no holes visiable. This shows that the fractal dimension of the Serpinski's gasket fractal in 3D is 2, that is, it has area but not volume.

Solution 6

Question 7

Below is data for the velocity of a river that represents measurements taken at different positions accross the river.

The data value $[x, y, v]$ means at position *x* meters the river is *y* meters deep and we measured the velocity of the water at position *x* at depth 40% above the river bed to be *v* meter per second. You can use v as the average velocity at position *x*.

The data value $[x, y, v_1, v_2]$ means at position *x* meters the river is *y* meters deep and we have measured the velocity to be v_1 $m \cdot s^{-1}$ 20% up from the river bed and v_2 $m \cdot s^{-1}$ 20% from the surface of the river. The average of

v_1 and v_2 would give an estimate for the average velocity at position x.

You are to estimate the total flow of the river in $m^3 \cdot s^{-1}$. Your Maple code should work for any data of this form. You may assume the first data point is for one side of the river and the last data point is the other side of the river and that there are at least 3 data points, so at least one measurement was taken.

Please also plot the cross section of the river and indicate where the measurements were taken with say vertical lines.

```
> Data :=
  [ [0,0.0,0.0],
    [5,0.2,0.1],
    [10,0.3,0.2],
    [18,0.3,0.3],
    [25,0.4,0.3,0.50],
    [32,0.6,0.35,0.55],
    [38,0.72,0.40,0.60],
    [43,0.6,0.30,0.60],
    [47,0.3,0.30],
    [50,0.0,0.0] ];
```

Solution 7

Question 8

The following is a solution to the random walk exercise in Assignment 2 by one of the students in the class. It is correct but very slow for large values of n.

```
> R := rand(1..4):
  Walk := proc(n) local x,y,i,P,Q;
    x := 0;
    y := 0;
    Q := [[0,0]];
    for i from 1 to n do
        P := R();
        if P=1 then x := x+1;
        elif P=2 then x := x-1;
        elif P=3 then y := y+1;
        else y := y-1;
        fi;
        Q := [op(Q),[x,y]];
    od;
    Q;
  end:
  plot( Walk(1000), style=line );
> Walk(3);
```

$$[[0, 0], [1, 0], [1, 1], [1, 2]]$$

To measure the efficiency of the Maple procedure we need to time it for different values of n. To time it use the Maple time(); command as follows

```
> start := time(); Walk(n): timetaken := time()-start;
```

Input the above program and time how long this code takes to execute for n=4000, n=8000, n=16000, n=32000.
Interpret the timings you get. Is the running time of the Walk procedure linear in n, quadratic in n, cubic in n, or something else?

The problem is the code Q := [op(Q),[a,b]] which appends the next point to the list . Doing this in a loop means that we are creating a list of length 2 then 3 then 4 then 5 and so on. Creating lists of length 1000 takes time. This makes the procedure slow. One solution is to store the points in an Array instead of a list. To create an Array of n values indexed from 1 to n use

```
> A := Array(1..n);
```

To insert a value into the Array use

```
> A[1] := [0,0];
```

At the end of the loop you still need to output a list of the points [A[1], A[2], ..., A[n]] which you can create in one step using the seq command like this: **A := [seq(A[i], i=1..n)];** Program this and retime the improved code for n=4000, n=8000, n=16000, n=32000, n=64000. It should make a huge difference.

Solution 8