Assignment 1, MACM 204, Fall 2013

Due Tuesday September 17th at 4:30pm at the beginning of class.
Late penalty: -20% for up to 24 hours late. 0 after that.

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There are 9 questions. Please attempt each question in a seperate worksheet (so that you don't destroy previous work). Print your Maple worksheets (you can use this small font and print double sided if you wish) and hand them in to me.

The purpose of this assignment is to get you to learn the basics of Maple: how to input formulas, how to graph functions, solve equations, calculate derivatives and integrals, and to use various kinds of loops (i.e. some basic programming).

▼ Question 1

Consider the function $f(x) = x \, e^x$. Construct a plot of $[\, f(x), f\,'(x), f\,''(x)\,]$ on the same graph for the domain $0 \leq x < 10$.

Read the help page for `?plot,options`. Using the appropriate options, create a legend for the plot, and add a suitable title.

► Solution 1

▼ Question 2

Consider the polynomials $f := x^4 - 1$ and $g := x^4 - 4\,x^3 + 8\,x - 4$.

Graph them on a suitable domain for x so that we can see which have real roots and which don't.

Now factor the polynomials. Why does the factor command not factor them into linear factors? Now, using **solve**, solve for the roots. Verify that the 4 roots of the polynomial g are correct. Now, using **fsolve**, solve for numerical approximations of the roots.

Read the help page for **?fsolve** to find out how to get the complex roots.
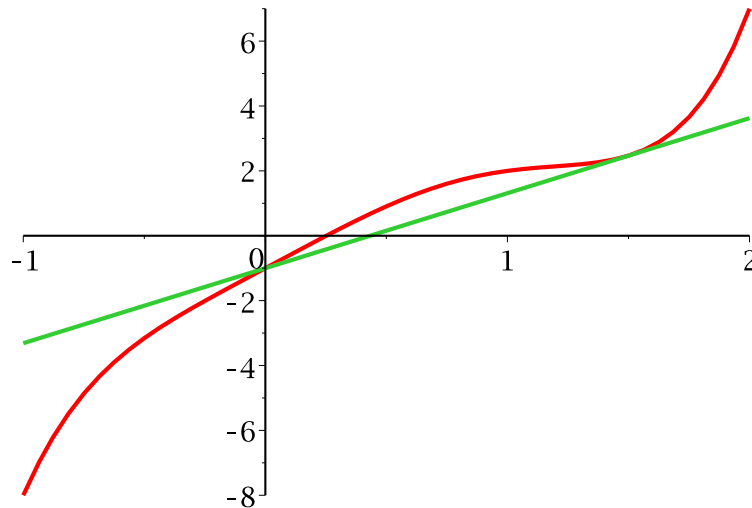
► Solution 2

▼ Question 3

Shown in the plot below is a plot of $f := x^5 - 2\,x^4 + 4\,x - 1$ on the domain -1 < x < 2. Shown also is the line tangent to $f(x)$ at $x = \dfrac{3}{2}$. Use Maple to reproduce the plot. Now express the area enclosed by the tangent line and $f(x)$ as a definite integral. Evaluate the definite integral using Maple.

```
> f := x^5-2*x^4+4*x-1;
```
$$f := x^5 - 2\,x^4 + 4\,x - 1$$

▼ Question 4

Consider the two equations $y^2 = x^3 + x$ and $x^2 + y^2 - 1$ which are the equations of an eliptic curve and a circle. Graph the two curves together using the **plots [implicitplot]** command. Improve the smoothness of the plot using the grid option if necessary. The circle will appear distorted. Find out from ?plot,options how to scale the axes so that the circle looks like a circle (so that the plot looks like the figure below). You can read off from the plot roughly where the curves intersect. Using the **fsolve** command compute the intersection points accurately.
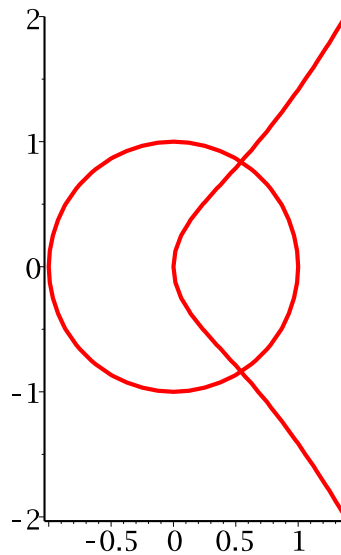
▼ Question 5

Calculate the following antiderivatives in Maple.

(i) $\int x^2 e^{-x} \, dx$ (ii) $\int x^{\frac{1}{2}} e^{-x} \, dx$ (iii) $\int x^2 \cos(x) \, dx$ (iv) $\int \cos(x) \, x^{\frac{1}{2}} \, dx$.

To check that the results are correct, use Maple to differentiate each antiderivative.
Now try to find an antiderivative that Maple cannot do.

▼ Question 6

Consider the definite integral $\int_0^\infty x^n e^{-x} \, dx$.

For $n = 1, 2, 3, 4, 5$ use Maple to calculate the integral. From these values deduce the formula for the integral as a function of n.
What do you get when you try to integrate the definite integral in Maple for n left as a symbolic parameter? Do you get the same result?
See the help page for **?GAMMA** .

The next three questions are to get you to write some loops and to experiment.

▼ Question 7

The first n terms of the Taylor series for e^x are
$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^n}{n!} \ .$$
Write a Maple **for loop** that constructs the Taylor polynomial for any value of n.
Run it for $n = 10$.

▶ Solution 7

▼ Question 8

The so called **3 x + 1** conjecture is the following.
If you start from any positive integer x, e.g., $x=9$, and you apply the following rule repeatedly
  if x is even, divide x by 2 else multiply x by 3 and add 1
you get 9, 28, 14, 7, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ...
You see that we end up at 1 then we get into a little cycle.
The conjecture is that for every positive integer x you ALWAYS get to 1. No one has proven it though!

Write a Maple **while loop** that generates the sequence for different values of x to get a feel for what happens. Try to find a value of x less than 50 that has a very long sequence.

► **Solution 8**

▼ Question 9

Peter Borwein has a fractal image on his home page [under Vis Number Theory if you care to look.] He created it by computing the complex roots of lots of polynomials then plotting them. The polynomials are special in that the coefficients are restricted to be +1, -1, only e.g. $x^6 + x^5 - x^4 - x^3 + x^2 - x - 1$. He simply graphed the complex roots in the complex plane and out popped a beautiful fractal image. It was totally unexpected. Try to do this in Maple.

To plot n points in the plane you can use the
`plot( [[x1,y1],[x2,y2],...,[xn,yn]], style=point, symbol=cross );`
command.
To compute the complex roots of a polynomial use the
`  R := [fsolve( poly, x, complex )];`
command.
If you have a complex number z, the Maple commands Re(z) and Im(z) give you the real and imaginary parts respectively. Hence to get a list of real and imaginary co-ordinates use `[seq( [Re(z),Im(z)], z=R )]`.
For example

```
> R := [fsolve(x^8+x^7-x^6-x^5+x^4-x^3-x^2-x+1,x,complex)];
```
$R := [-1.312377860 - 0.5783661599\,I,\ -1.312377860 + 0.5783661599\,I,$
$\quad -0.5357957822 - 0.6923670476\,I,\ -0.5357957822 + 0.6923670476\,I,$
$\quad 0.5000000000 - 0.8660254038\,I,\ 0.5000000000 + 0.8660254038\,I,$
$\quad 0.5565090232,\ 1.139838261]$

```
> P := [seq( [Re(z),Im(z)], z=R )];
```
$P := [[-1.312377860,\ -0.5783661599],\ [-1.312377860,\ 0.5783661599],\ [$
$\quad -0.5357957822,\ -0.6923670476],\ [-0.5357957822,\ 0.6923670476],$
$\quad [0.5000000000,\ -0.8660254038],\ [0.5000000000,\ 0.8660254038],$
$\quad [0.5565090232,\ 0.],\ [1.139838261,\ 0.]]$

```
> plot( P, style=point, symbol=cross, axes=box );
```
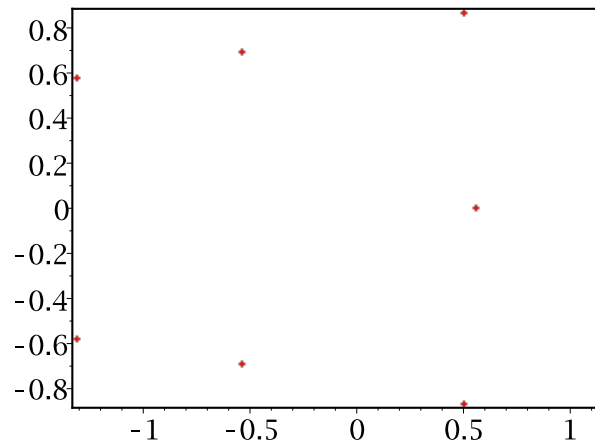
Find a way to generate lots of polynomials with coefficients 1, or -1 only (Peter did this for polynomials of degree 18), compute their complex roots and plot them on the same graph. To get a good picture, you will need 500 or more polynomials. The following Maple procedure might be useful - it generates +1, -1 at random.

```
> C := proc() if rand() mod 2 = 1 then 1 else -1 fi end:
  for i to 5 do C() od;
```

$$1$$
$$-1$$
$$-1$$
$$-1$$
$$1$$

► Solution 9