MACM 202 Assignment 4, Fall 2005

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This assignment is worth 10% of your grade. It is due Tuesday November 8th at 12 noon. A late penalty of 20% will apply for each day late. Attempt all questions. For the exercises in the text use Maple where appropriate.

Question 1 (20 marks)

Consider again Serpinski's gasket as shown on page 112. In question 4 of assignment 3 you were asked to draw Serpinski's gasket in 2 dimensions using two methods. One, a direct recursive construction, the other, a random walk, described on pages 121–124 as the "Fractal game". Generalize both of these to 3 dimensions. In three dimensions you will start with a regular tetrahedron instead of an equilateral triangle. A regular tetrahedron has four vertices p, q, r, s. One choice is $p = (1, 1, 1), q = (-1, -1, 1), s = (-1, 1, -1), r = (1, -1, -1)$.

Note, the `POINTS(...)`, `CURVES(...)`, `POLYGONS(...)` graphics primitives in Maple understand lists of the form `[x,y,z]` to mean the point (x, y, z) in 3 dimensions. See the Maple worksheet `graphics.mws` for examples.

Question 2 (20 marks)

Let n be a positive integer. The number theoretic function $\lambda(n)$ is defined as follows.

$\lambda(n) = -1$ if n has an odd number of prime divisors

$\lambda(n) = +1$ if n has an even number of prime divisors.

Thus $\lambda(10) = \lambda(2 \times 5) = +1$ and $\lambda(12) = \lambda(2 \times 2 \times 3) = -1$. Write a Maple procedure to compute $\lambda(n)$. Now write a Maple procedure to construct a "random walk" for $\lambda(n)$ as follows. The walk will consist of a sequence of points $u_0, u_1, u_2, u_3, ...$ starting with $u_0 = [0, 0]$ and $u_1 = [1, 0]$. For $n = 2, 3, 4, 5, ...,$ if $\lambda(n) = -1$ turn clockwise and take a step of length 1, otherwise, if $\lambda(n) = +1$, turn anti-clockwise and take a step of length 1. Thus the next few points are $u_2 = [1, -1]$, $u_3 = [0, -1]$, and $u_4 = [0, -2]$. Generate a Maple plot of the form

PLOT(CURVES($[u_0, u_1, u_2, ..., u_N]$), SCALING(CONSTRAINED))

for $N = 3000$.

Questions From the Text (60 marks)

Do exercises 4.2, 4.4, 4.9, 4.14, and 4.18 from the text.

For question 4.2 use the **fit** command from the `fit.mws` worksheet or the `LeastSquares` command from the *CurveFitting* package, to find the linear function and then the exponential function that fit the data in the least-squares sense. Then plot the data and both functions that you obtain on the same graph in Maple, and, by eye, state which is the best fit. Note, to fit an exponential function of the form $x = ce^{kt}$, to some (t_i, x_i) data first note that $\log x = \log c + kt$, hence, if you fit a straight line to $(t_i, \log x_i)$ data, you will obtain $\log c$ and k from which you can recover c. A worked example of how to do this is shown in the fit.mws worksheet.

For question 4.9, there are three unknowns to determine, a, b and γ. The book is suggesting that you use (i) N(0), (ii) N(∞), and (iii) N($t_{1/2}$) which you may read off from the plot, i.e., you are not being asked to do a least-squares fit to the data.

For question 4.18 generate also plots which illustrate the qualitative behaviour for $m = 1$ and $m = 2$.