MACM 401 MATH 701 MATH 819 CMPT 881
Assignment 6, Spring 2013.

Michael Monagan

This assignment is to be handed in by Monday April 15th 10am.
For problems involving Maple calculations and Maple programming, you should submit a printout of a Maple worksheet of your Maple session.
Late Penalty: -20% for up to 24 hours late. Zero after that.
Also, study examples 12.2, 12.3, 12.4 and 12.5 and the proofs of Theorem 12.3 and Theorem 12.4 (Liouville's theorem – the special cases only).

Question 1 (20 marks): Symbolic Integration

Implement a Maple procedure `INT` (you may use `Int` if you prefer) that evaluates antiderivatives $\int f(x)\mathrm{d}x$. For a constant c and positive integer n your Maple procedure should do the following

$$\int c\,dx = cx.$$

$$\int cf(x)\,dx \to c\int f(x)\,dx.$$

$$\int f(x)+g(x)\,dx \to \int f(x)\,dx + \int g(x)\,dx.$$

$$\text{For } c \neq 1 \quad \int x^c\,dx = \frac{1}{c+1}x^{c+1}.$$

$$\int x^{-1}\,dx = \ln x.$$

$$\int e^x\,dx = e^x \quad \text{and} \quad \int \ln x\,dx = x\ln x - x.$$

$$\int x^n e^x\,dx \to x^n e^x - \int nx^{n-1}e^x\,dx.$$

You may ignore the constant of integration. NOTE: e^x in Maple is `exp(x)`, i.e. it's a function not a power. HINT: use the `diff` command for differentiation to determine if a Maple expression is a constant wrt x. Test your program on the following.

```
> INT( x^2 + 2*x + 1, x );
> INT( x^(-1) + 2*x^(-2) + 3*x^(-1/2), x );
> INT( exp(x) + ln(x) + sin(x), x );
> INT( 2*f(x) + 3*y*x/2 + 3*ln(2), x );
> INT( x^2*exp(x) + 2*x*exp(x), x );
> INT( 2*exp(-x) + ln(2*x+1), x );
```

Question 2: Non-elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to prove that the following integrals are not elementary.

$$\int \frac{e^x}{x}\mathrm{d}x$$

$$\int \frac{1}{\log(x)^2}\mathrm{d}x$$

$$\int \frac{\log(x)}{x+1}\mathrm{d}x$$

$$\int e^x \log(x)\mathrm{d}x$$

Question 3: Elementary Integrals (20 marks)

Reference: 12.6, 12.7

Apply the Risch algorithm to compute the following elementary integrals.

$$\int \frac{e^{2x}}{e^{2x}+e^x+1}\ \mathrm{d}x$$

$$\int 2\theta + 2 - \frac{1/x+1}{(\theta+x)^2} + \frac{1}{x\theta}\ \mathrm{d}x \qquad \text{where } \theta = \log(x)$$

$$\int \theta + x\theta + \frac{2}{x}\theta^2 - \frac{1}{x^2}\theta^2\ \mathrm{d}x \qquad \text{where } \theta = e^x$$

Question 4: Polynomial Resultants (10 marks)

Consider the following pairs of polynomials

$$f_1 = 2x^3 + 2x - x^2 - 1,\ g_1 = x^2 - 2$$

$$f_2 = (x^2 + 1 - x)z + (x^4 + 2\,x^2 - x),\ g_2 = z^2 + 1 \text{ and}$$

$$f_3 = 2yx^3 - x^2y^2 + (2y+1)x - 3,\ g_3 = 2x^2 - (3-y^2)x + (2y^2 - 5).$$

Compute the resultants $\texttt{res}_x(f_1, g_1)$, $\texttt{res}_z(f_2, g_2)$ and $\texttt{res}_x(f_3, g_3)$ (please note which variable is being eliminated!) using the Maple command `resultant` (so that you know what the answers are). In class I showed how we can modify the natural Euclidean algorithm to compute the resultant. Program this in Maple. Execute it on the above inputs. Now modify that code to use the *primitive* Euclidean algorithm to compute the resultant. Execute it on the above inputs.